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An Improved Quantum State Estimation algorithm via Compressive Sensing

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***Abstract*—The dimension of the density matrix of a quantum system increases with the qubits of the quantum system, which makes the quantum state estimation time consuming and requires huge computation. In order to reduce the computational time in quantum state estimation, the problem of quantum state estimation based on compressive sensing is changed to the optimization problem with error constraint. In this paper, an improved Alternating Direction Method of Multipliers (ADMM) algorithm is proposed to design the optimization scheme of solving the pure state of quantum state estimation in the cases of with and without external noise. The experiments are implemented in the MATLAB. The comparison results between adaptive and fixed weight value indicate that the improved algorithm has better performances in both aspects of estimation accuracy and robustness to external disturbances. We also extend the quantum state estimation to the qubits of six and seven.**

I. INTRODUCTION

THEquantum state can be described by a density matrix, whose entries can be estimated or reconstructed by the statistical measurements from experiments. This process of quantum state estimation or reconstruction is similar to the medical X-ray tomography initially, so one also calls it the quantum tomography[1]. In recent years, compressive sensing has paid widely attention, which is a new method of reconstructing signal by less measurements and it has been applied to the quantum state estimation[2-4].

In quantum state estimation, compressive sensing can be used to get low dimensional measurements from a projection of high dimensional density matrix, and reconstruct the density matrix accurately. Thereby only less measurements are needed to recognize the whole variables comparing to the quantum tomography method. Since the density matrix of the pure quantum state is a Hermitian matrix, theoretically only half of the elements are needed to estimate the full density matrix based on matrix transformation[5]. While using compressive sensing, one needs to get the values of the density matrix of pure states by even much less than half amount of measurements[6]. Because the number of measurements is less than the number of elements of density matrix, one only can obtain all elements by solving the optimization problem. Therefore, the process of quantum state estimation becomes an optimization problem. There are some common optimization algorithms used to estimate quantum state. Smith[7] summarized the least square problem and solved it in quantum state estimation through MATLAB toolbox. Liu[8] adopted Dantzig algorithm to estimate the density matrix. In recent years, the Alternating Direction Method of Multipliers (ADMM) algorithm has got the attention of researchers[9] due to its fast computation and good robustness. This method reconstructs the quantum state by minimizing the nuclear norm of the density matrices[10]with the prior information. Li[11] applied ADMM algorithm to quantum tomography based on compressive sensing and gave the formation of the optimization problem, finally the experiments acquire a more accurate estimation of a density matrix with 5 qubits.

In this paper, we reconstruct the density matrix with 6 qubits based on compressive sensing by using ADMM algorithm. Simulation results are given and analyzed by comparing estimation errors with and without external noise under different measurement rates. The result shows that ADMM algorithm can obtain better performance of the quantum state estimation with short convergence time and high accuracy.

In this paper we denote *argmin* $f(x)$ as the value of variable x when function $f(x)$ has the minimum value. $mat(\cdot)$ is the operator to convert a vector to a matrix. $vec(\cdot)$ represents the transformation from matrix to vector. $tr(\cdot)$ is the operator to calculate the trace. $abs(\cdot)$ means absolute value. $\max(\cdot)$ means maximum value. $\|\cdot\|_F$ is the nuclear norm. $\|\cdot\|_F$ represents the Frobenius norm.

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II. QUANTUM STATE ESTIMATION BASED ON COMPRESSIVE SENSING

A. Compressive Sensing

The core idea of compressive sensing is to randomly project the original sparse signal x to obtain a small number of measured values y. By using the priori information of the sparse signal x, the matrix can be reconstructed through optimization methods, specifically by minimizing

$$\hat{x} = \arg\min \|x'\|_* \quad \text{s.t. } \mathbf{y} = \phi x \tag{1}$$

where $x \in \mathbb{C}^{N\times N}$ is the original signal, $\mathbf{y} \in \mathbb{C}^{M\times 1}$ is the measured vector, which is the liner measurement of x, $\phi : x \in \mathbb{C}^{N\times N} \to y \in \mathbb{C}^{M\times 1}, \phi \in \mathbb{C}^{M\times N^2}$ is the sample matrix, $\hat{x}$ is the reconstruction signal, x' is the transformation of original signal x, which $x' = \Phi x$, and Φ is the unitary matrix, which means $\Phi^* \Phi = \Phi \Phi^* = I_N$. $\|\cdot\|_*$ is the nuclear norm, $\|x'\|_* = tr(\sqrt{x'^* x'}) = \sum_{i=1}^{\min\{m,n\}} \sigma_i$, the rank of x' can be minimized by minimizing the nuclear norm. In this way we can rewrite equation (1) as:

$$\hat{x} = \arg\min \|\Phi x\|_* \quad \text{s.t. } \mathbf{y} = \phi \Phi^T x' \tag{2}$$

The dimension of $\mathbf{y}$ is M which is much less than that of $x : M << N^2$. The measurement is obtained from random projecting signal x to the selected Mth rows from the sample matrix $\phi = \{\phi_i\}_{i=1}^M$ to get the measured vector $\{y_i\}_{i=1}^M$.Therefore the Mth inner product value $y_i = \langle x, \phi_i \rangle$ can be calculated between x and $\{\phi_i\}_{i=1}^M$ to form the measured vector $\mathbf{y} = (y_1, y_2, ..., y_M)$.The size of the sample matrix ϕ directly decides the compressive rate M/N^2 of compressive sensing, which is also called the measurement rate η.

In fact, it is inevitable to introduce the measurement noise, which results in the error ε, so equation (2) may be rewritten as

$$\hat{x} = \arg\min \|\Phi x\|_* \quad \text{s.t. } \|\mathbf{y} - \phi \Phi^T x'\| < \varepsilon \tag{3}$$

where the value of ε is the expectation error, which is related directly to the selection of Φx.

The sample matrix ϕ of compressive sensing should meet certain conditions. The observation matrix and the signal matrix must not be relevant, which is equivalent to satisfy the Restricted Isometry Property (RIP):

$$(1-\delta)\|x\|_F \le \|\phi x\|_2 \le (1+\delta)\|\mathrm{x}\|_F \tag{4}$$

where $\delta \in (0,1)$ is a constant.

When the sample matrix ϕ meets the RIP, one can use measured vector $\mathbf{y}$ to reconstruct the signal x ,which is to solve a problem of N^2 unknown variables from M equations. Because $M \ll N^2$, the equations may have countless solutions. Fortunately original signal x can be reconstructed by solving an optimization problem which minimizes the nuclear norm of x' due to the prior information, so a unique solution of x can be achieved.

B. Problem Description of Quantum Tomography based on Compressive Sensing

In a quantum system, the density matrix is $\rho \in \mathbb{C}^{d\times d}$, $\rho = \sum_{i=1}^r p_i |\psi_i\rangle\langle\psi_i|$, $|\psi_i\rangle$ is wave function, p_i is the probability of the wave function $|\psi_i\rangle$, $\boldsymbol{\psi}$ is the vector made by $|\psi_i\rangle$, $\boldsymbol{\psi} = [\sqrt{p_1}|\psi_1\rangle, \sqrt{p_2}|\psi_2\rangle, ..., \sqrt{p_r}|\psi_r\rangle]$, $\boldsymbol{\psi} \in \mathbb{C}^{d\times r}$, $\rho = \boldsymbol{\psi} \cdot \boldsymbol{\psi}^*$, so the rank of density matrix is at most r due to the rank property of multiplication of two matrices. The dimension of n qubit density matrix is $d = 2^n$, when the pauli matrices are selected as orthogonal bases.

The observation matrix $\mathbf{O}^*$ of quantum state estimation can be calculated as:

$$\mathbf{O}^* = \sum_{i1,...,in=0}^{3} \sigma_{i1} \otimes \sigma_{i2} \otimes \cdots \otimes \sigma_{in} \tag{5}$$

where $\sigma_{i1}, \sigma_{i2}, ..., \sigma_{in}$ can be represented by a unit matrix I and pauli matrices $\sigma_1, \sigma_2, \sigma_3$, $I = \sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\sigma_2 = \begin{bmatrix} 0 & \text{-i} \\ \text{i} & 1 \end{bmatrix}$, $\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

Define the system measurement operator A as A : $C^{d\times d \to M}$, the sample matrix is $\mathbf{A} \in \mathbb{C}^{M\times d^2}$, which randomly select Mth rows from observation matrix $\mathbf{O}^* \in \mathbb{C}^{d^2 \times d^2}$, so the ith row of sample matrix $\mathbf{A}$ is the ith row of observation matrix $\mathbf{O}^*$. The sample matrix $\mathbf{A}$ and observation matrix $\mathbf{O}^*$ meets the RIP. The process of measurement is to randomly project $\hat{\rho}$ to the sample matrix to obtain less measurements y_i. Mth measured values y_i constitute measured vector $\mathbf{y}$, which can be represented as:

$$y_i = (\mathrm{A}(\hat{\rho}))_i + e_i = c \cdot tr\left(\mathbf{O}_i^* \hat{\rho}\right) + e_i,\ i = 1, ..., M \tag{6}$$

$$\mathbf{y} = \mathbf{A} \cdot vec(\rho) + \mathbf{e} \tag{7}$$

where $\mathbf{A} \in \mathbb{C}^{M\times d^2}$, $\mathbf{y} \in \mathbb{C}^{M\times 1}$, $\mathbf{e} \in \mathbb{C}^{M\times 1}$, $\mathbf{O}^* \in \mathbb{C}^{d^2\times d^2}$, $\hat{\rho} \in \mathbb{C}^{d\times d}$, $vec(\rho) \in \mathbb{C}^{d^2\times 1}$, c is a normalization parameter. $\mathbf{e}$ is the measurement error or external system noise. The normalized error is

$$error = \left\|\rho^* - \hat{\rho}\right\|_2^2 \Big/ \left\|\rho^*\right\|_2^2 \tag{8}$$

ρ^* is given by the following formula [12]:

$$\rho^* = \frac{\boldsymbol{\psi} \cdot \boldsymbol{\psi}^*}{tr\left(\boldsymbol{\psi} \cdot \boldsymbol{\psi}^*\right)} \tag{9}$$

where ρ^* is the true density matrix randomly generated, the estimation error is the normalization error of the difference between the true density matrix ρ^* and the estimated density matrix $\hat{\rho}$, which is treated as one of the quality to measure the state estimation.

When there is an external noise, the average system measured values is expressed as:

$$y_i = (\mathrm{A} \cdot (\hat{\rho} + \mathbf{S}))_i + e_i = c \cdot tr(\mathbf{O}_i^* (\hat{\rho} + \mathbf{S})) + e_i \tag{10}$$

$$\mathbf{y} = \mathbf{A} \cdot vec(\rho + \mathbf{S}) + \mathbf{e} \tag{11}$$

where $\mathbf{S} \in \mathbb{C}^{d \times d}$ is sparse matrix, $i = 1, ..., M$.

In this paper, wedefine the system measurement rate η as:

$$\eta = \frac{M}{d^2} \tag{12}$$

where M means the sample matrix randomly selecting Mth rows from $\mathbf{O}^*$, d^2 is the dimension of $\mathbf{O}^*$. Measurement rate η denotes the percentage of the sample matrix $\mathbf{A}$ occupied observation matrix $\mathbf{O}^*$.

C. Quantum State Tomography based on Compressive Sensing

Quantum state estimation is a method based on statistical information to reconstruct the density matrix of quantum systems through measuring the same unknown quantum states many times. The quantum tomography computation is very cost due to the large number of variables. Because compressive sensing can recover sparse/low rank signals with less measurements,it is natural to combine compressive sensing and quantum tomography to estimate quantum states with shorter computation time. The stateonehope to reconstruct can be described as a density matrix which can be optimized through:

$$\arg\min_{\rho} \|\rho\|_* \ \text{ s.t. } \mathbf{y} = \mathbf{A} vec(\rho) \tag{13}$$

where $\rho \in \mathbb{C}^{d \times d}$ is the density matrix, denote the quantum state, d is the dimension of density matrix ρ ,which is equal to N in the dimension of x, $\mathbf{A} \in \mathbb{C}^{M \times d^2}$ is the sample matrix equaled to ϕ, which is randomly selected Mth rows from the observation matrix $\mathbf{O}^* \in \mathbb{C}^{d^2 \times d^2}$, $\mathbf{y} \in \mathbb{C}^{M \times 1}$ is the measured vector. In this paper, density matrix ρ is the original signal x and $\mathbf{A}$ is the sample matrix ϕ, so the target is to use less measured values to reconstruct ρ.

There are two advantages for this method: first, it only needs a small numberof random measurements of the original signal. Second, regarding the lost part of the original signal and the external noise, this method has good robustness.

III. Improved Algorithm Optimization and Application

A. ADMM Algorithm

In order to get good estimation, we propose an improved ADMM algorithm with adaptive weight value, which can be described as follows:

$$\text{minimize } f(\mathbf{x}) + g(\mathbf{z}) \quad \text{s.t.} \mathrm{A}\mathbf{x} + \mathrm{B}\mathbf{z} = \mathbf{c} \tag{14}$$

where $\mathbf{x}$, $\mathbf{z} \in \mathbb{R}^n$, $f(\mathbf{x})$ and $g(\mathbf{z})$ are optimization functions. A Lagrangian is introduced to change the optimization problem with constraints into the optimization problem without constraint. The Lagrangian form of the algorithm is:

$$\begin{aligned} L_\lambda(\mathbf{x}, \mathbf{z}, \mathbf{y}) &= f(\mathbf{x}) + g(\mathbf{z}) + \mathbf{y}^T (\mathrm{A}\mathbf{x} + \mathrm{B}\mathbf{z} - \mathbf{c}) \\ &+ \frac{\lambda}{2} \|\mathrm{A}\mathbf{x} + \mathrm{B}\mathbf{z} - \mathbf{c}\|_2^2 \end{aligned} \tag{15}$$

in which, $\lambda > 0$ is the weight value, $\mathbf{u} = \mathbf{u}' / \lambda$ .The estimation error of the ADMM algorithm depends on the weight value λ. However, the weight value λ is fixed which may be too large or too small to get a good estimation. So we propose here an improved method of adaptive weight value λ, which can be described as:

$$\lambda^{\mathrm{k}+1} = \begin{cases} 1.05\lambda^{\mathrm{k}} & error^{\mathrm{k}} < error^{\mathrm{k-1}} \\ 0.7\lambda^{\mathrm{k}} & error^{\mathrm{k}} > error^{\mathrm{k-1}} \\ \lambda^{\mathrm{k}} & others \end{cases} \tag{16}$$

The optimal solution then can be obtained by the iteration of the following three steps:

a) $\mathbf{x}$ minimization: $\mathbf{x}^{k+1} = \arg\min_{\mathbf{x}} L_\lambda(\mathbf{x}, \mathbf{z}^k, \mathbf{y}^k)$

b) $\mathbf{z}$ minimization: $\mathbf{z}^{k+1} = \arg\min_{\mathbf{z}} L_\lambda(\mathbf{x}^{k+1}, \mathbf{z}^k, \mathbf{y}^k)$

c) $\mathbf{y}$ update: $\mathbf{y}^{k+1} = \mathbf{y}^k + \lambda(\mathbf{A}\mathbf{x}^{k+1} + \mathbf{B}\mathbf{z}^{k+1} - \mathbf{c})$

B. Application in Quantum State Estimation

In this section, we combine the ADMM algorithm with compressive sensing to estimate the quantum state. The problem of quantum sate estimation can be described as:

$$\hat{\rho} = \arg\min_{\rho} \|\rho\|_* \quad \text{s.t.} \|\mathbf{y} - \mathbf{A}vec(\rho)\|_2^2 \le \varepsilon \tag{17}$$

where $\varepsilon > 0$ is the expected error of state estimation. The nuclear norm of $\|\rho\|_*$ is a function which can be optimized effectively. By minimizing the nuclear norm $\|\rho\|_*$, one can estimate density matrix by means of compressive sensing.When estimation error is less than the expected error ε, optimization is over and $\hat{\rho}$ is the estimation of ρ^*.

During the measurement, the noise introduced by external environment interferes the estimation accuracy of system. Usually the noise is assumed to satisfy a distribution, such as Gaussian noise. When external noise $\mathbf{S}$ can't be ignored, one can get estimation of true density matrix ρ^* by minimizing:

$$\begin{aligned} &\text{minimize } \|\rho\|_* + \|\mathbf{S}\|_1 \\ &\text{s.t.} \|\mathbf{y} - \mathbf{A}vec(\rho + \mathbf{S})\|_2^2 \le \varepsilon \end{aligned} \tag{18}$$

where sparse noise $\mathbf{S} \in \mathbb{C}^{d \times d}$ is added directly on density matrix ρ and the dimension of $\mathbf{S}$ is the same as ρ. Therefore one can get two unrelatedvariable sets, which both satisfy RIP. The Lagrange form is

$$\begin{aligned} L_\lambda(\rho, S, \mathbf{u}') = &\|\rho\|_* + \|\mathbf{S}\|_1 + \mathbf{u}'^T \left(\mathbf{A}vec(\rho)\right. \\ &\left.+\mathbf{A}vec(\mathbf{S}) - \mathbf{y}\right) + \frac{\lambda}{2}\|\mathbf{A}vec(\rho) + \mathbf{A}vec(\mathbf{S}) - \mathbf{y}\|_2^2 \end{aligned} \tag{19}$$

where λ is a weight value, which effect the convergence rate and the number of iterations. We then combine the liner withthe quadratic to get:

$$\begin{aligned} L_\lambda(\rho, S, \mathbf{u}) = &(\|\rho\|_* + \|\mathbf{S}\|_1 \\ &+ \frac{\lambda}{2}\|\mathbf{A}vec(\rho) + \mathbf{A}vec(\mathbf{S}) - \mathbf{y} + \mathbf{u}\|_2^2) \end{aligned} \tag{20}$$

where $\mathbf{u} = \mathbf{u}'/\lambda$, parameter $\lambda > 0$.

The iteration consists of the following three steps:

a) ρ minimization: $\rho^{k+1} = \arg\min_{\rho} L_\lambda(\rho, \mathbf{u}^k, \mathbf{y}^k)$

b) $\mathbf{S}$ minimization: $\mathbf{S}^{k+1} = \arg\min_{\mathbf{S}} L_\lambda(\rho^{k+1}, \mathbf{S}^k, \mathbf{u}^k)$

c) $\mathbf{u}$ update: $\mathbf{u}^{k+1} = \mathbf{u}^k + \lambda(\mathbf{y} - \mathbf{A}vec(\rho^{k+1}) - \mathbf{A}vec(\mathbf{S}^{k+1}))$

IV. PERFORMANCE COMPARISON OF ADMM OPTIMIZATION ALGORITHMS

A. Estimation Performance Comparison and Analysis under Fixed and Adaptive weight values

In this section we validate the superiority and robustness of ADMM algorithm based on compressive sensing of quantum state estimation.ADMM algorithm is analyzed from three aspects: 1) Improvement: get the best performance through fixed and adaptive weight value 2)Robustness: with and without external noise. 3) Superiority: find the best applicable condition under different quantum qubit.

1)Comparison of Different Fixed Weight Values: In order to find a fixed weight value λ to get the minimum estimation error under measurement rate η=0.4 at $n = 6$, the fixed λ select 10 different values from $\lambda = 0.1$ to $\lambda = 1.0$, $\Delta\lambda = 0.1$.Fig.1 shows the normalized estimation error under different fixed weight value λ, in which each of the histogram represent an estimation error decreasing of a fixed λ, from which one can see that: the estimation error gets the minimum $error = 0.1240$ with λ=0.3 among 10 different fixed weight values.One can conclude that neither too large nor too small fixed λ can get the least estimation error. So to find a method depending less on the value of λ is important, which brings out the adaptive weight value. In contrast, we choose $\lambda_{fixed} = 0.3$ as the best fixed weight value to compare with the adaptive ones.

2) Comparison between Adaptive and Fixed Weight Value: We compare estimation error between different initial adaptive weight values and fixed weight value $\lambda_{fixed} = 0.3$ .Fig.2 shows the estimation error between several different initial values of adaptive and fixed weight value under measurement rate η=0.4, from which one can see that: when the initial value of adaptive weight values are $\lambda = 1.0$, $\lambda = 2.5$ and $\lambda = 4.0$, respectively, the estimation error are $error$=0.1436, $error$=0.1168 and $error$=0.1436,

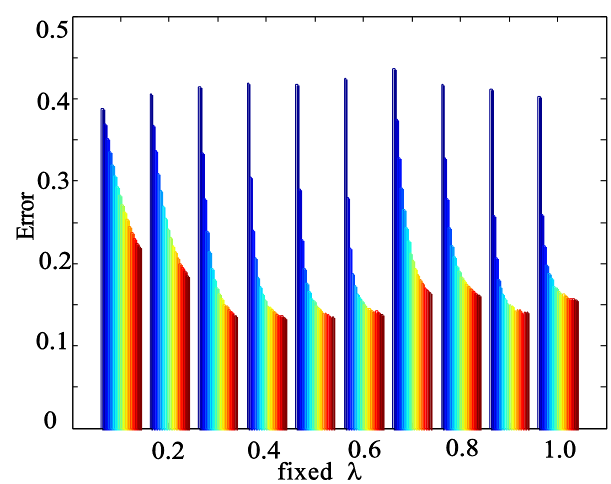

Fig.1 Estimation error under different fixed λ

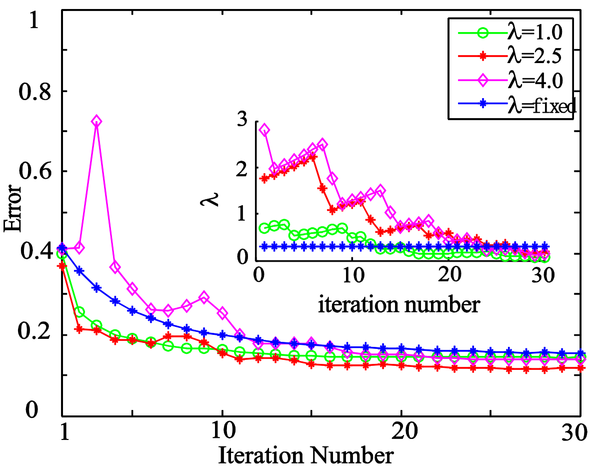

Fig.2 Estimation error under adaptive and fixed λ

respectively, in which the initial value $\lambda = 2.5$ gets the minimum estimation $error$=0.1168 . Meanwhile the estimation error of λ_{fixed} =0.3 is only $error$=0.1528 .

Fromcomparison between the fixed and adaptive weight value one can see that:

First, the estimation error of adaptive initial weight value $\lambda = 2.5$ rapidly decreased to $error = 0.2009$ only in 2 iterations, while the estimation error of fixed λ_{fixed} =0.3 uses at least 10 iterations to slowly decrease to $error = 0.2050$, which cost much more computation time.

Second, the estimation error of fixed λ_{fixed} is the largest among four values , which estimation accuracy is Δ=88.32%−84.72%= 3.6%less than adaptive $\lambda = 2.5$. In addition, all the estimation accuracy using adaptive weight value is higher than the fixed one. So the initial valuesaffect little in adaptive methods.

To sum up, the use of adaptive weight value get faster decline in the estimation error in less iteration, which can achieve high estimation accuracy at very short convergence time.

B. Estimation Performance Comparison and Analysis withoutand with Noise

We can conclude from obvious section that if one chooses the appropriate fixed weight value λ , the estimation error of fixed λ is close to the adaptive weight value. So in this section we choose the best fixed λ=0.3 to analyze the performance of ADMM algorithm by comparing the estimation error with and without noise at $n = 6$ .The measurement rate ranges from η=0.1 to η=0.5 and the change step is $\Delta\eta$=0.05 .

1) Without Noise: In the experiment,the iteration of ADMM algorithm is 30. The experiment results of the normalized estimation error with different measurement rate η are shown in Fig.3.

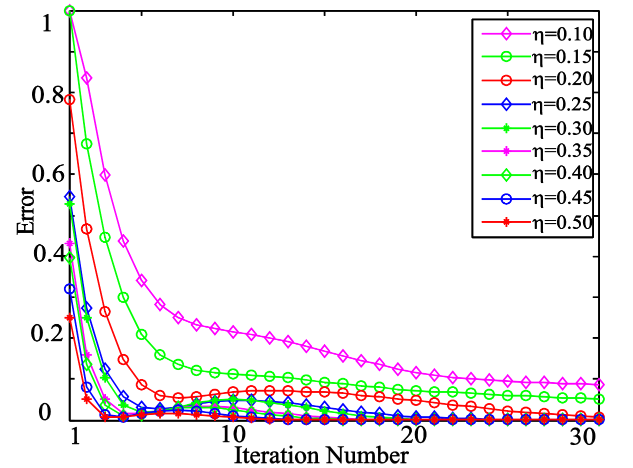

Fig.3Estimation error withoutnoise under different measurement rates

From which one can see that: when η=0.3 , the estimation error is $error$=0.0016 , which means the accuracy is 99.84%. With the increase of measurement rate from η=0.1 to η=0.5 , the estimation error changes from $error$=0.0852 to $error$=1.46e^{-6} , which estimation accuracy changes from 91.48% to 99.9999%. So the larger the measurement rate η , the smaller the estimation error. In addition, the estimation error decreased rapidly in the first 5 iterations. Therefore a good estimation can be obtained in no more than 10 iterations.

2) With External Noise: Due to the external noise caused by environments and measurement instruments. The robustness of improved ADMM algorithm is investigated. The number and value of external noiseare selected 10% of $\mathbf{O}^*$ which are $S_{number} = 0.01 * 4^6 \approx 41$ and $S_{size} = \pm 0.01$ respectively. The experiment results of the normalized estimation error under different measurement ratesare shown inFig.4, from which one can see that:

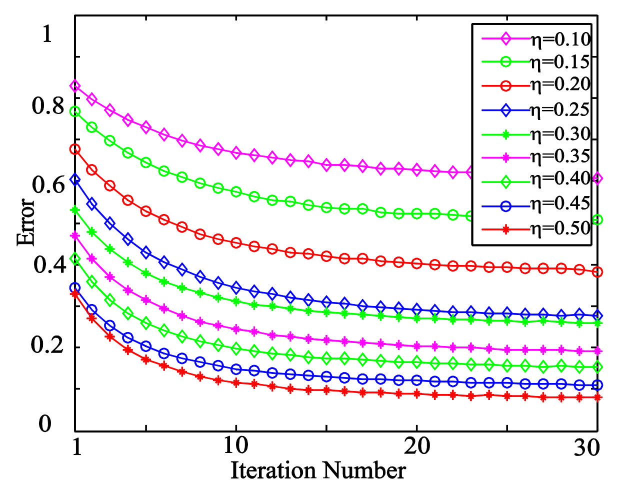

Fig.4 Estimation error with external noise under different measurement rates

First, the best estimation performance is obtained when the measurement rate is $\eta=0.5$. The estimation error of ADMM is $error=0.0798$.

Second, as the measurement rate increases from $\eta=0.2$ to $\eta=0.5$, the estimation accuracy improves significantly from 66% to 92.02%, which improved $\Delta=92.02\%-66\%=26.02\%$.

Third, the estimation error gradually decreases with iteration increasing. It needs at least 15 times to achieve estimation accuracy above 90% under $\eta=0.5$, which is 5 times more than that without noise. Therefore with external noise the computation time is longer than the case without noise.

To sum up, the estimation error decreased quickly and significantly without external noise, while the estimation error decreased gradually with external noise. When the estimation accuracy is fixed, the higher the measurement rate η, the less the iteration times, while when the measurement rate η is fixed, the more the iteration times, the higher the estimation accuracy.

C. Estimation Performance Comparison and Analysis under DifferentQuantum Qubits

Theoretically, when quantum qubit increases, the elements of density matrix dramatically increases. The dimension of n qubit density matrix is $d=2^n$. When quantum qubit is $n=5$, the elements of density and observation matrices are $d\times d=2^5\cdot 2^5=1024$ and $d^2\times d^2=4^5\cdot 4^5\approx 1.05\times 10^6$,respectively. As the quantum qubit increase to $n=7$, the elements of the densityand observation matrices increase significantly to $2^7\cdot 2^7=16384$ and $4^7\cdot 4^7\approx 2.68\times 10^8$. Fig. 5 are the experiment results of estimation error with external noise under qubit $n=5$, 6 and 7, from which one can see that: when the number of qubit increases from $n=5$ to $n=7$ under measurement rate $\eta=0.5$, the estimation error decreases from $error=0.1831$ to $error=0.0489$, which

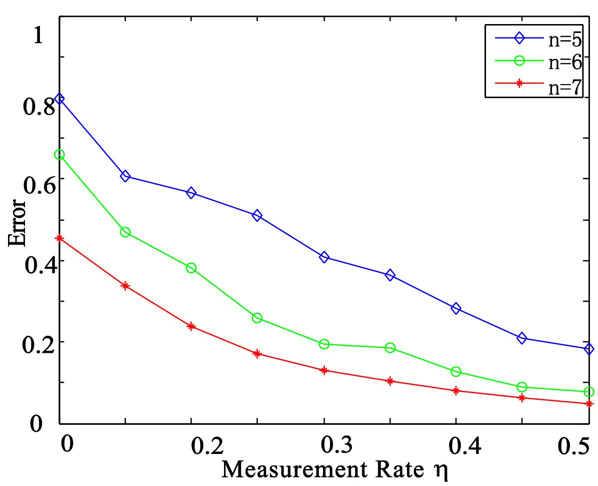

Fig.5 Estimation error underdifferent qubitwith noise

reduced $\Delta=0.1342$. In the meanwhile, the accuracy increases from 81.69% to 95.11%, which improved $\Delta=13.42\%$. If the requirement of estimation accuracy is fixed above 90%, the estimation accuracy of qubit $n=7$ first meets the requirement with measurement rate $\eta=0.35$, while the measurement rate of qubit $n=6$ should be $\eta>0.45$. So one can conclude that the larger the quantum qubit, the smaller the estimation error with the same measurement rate η.

V. Conclusion

In this paper we estimated quantum system states based on the theory of compressive sensing by using the ADMMoptimization algorithms.Experimental results indicated that by using compressive sensing to estimate high dimensional density matrix containing a large amount of elements, especially when the qubit of the system is larger, the ADMM algorithm can obtain small estimation error and have high robustness.

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